Feuille d'exercice n° 20 : **Analyse asymptotique – quelques corrigés**

**Exercice 14 ( ✎ )**

**1)** $0$

**2)** $\dfrac{1}{6}$

**3)** $\mathrm{e}^{\mathrm{e}^{-1}}$

**4)** $\mathrm{e}^{-1}$

**5)** $\dfrac{2}{3}$

**6)** $\dfrac{a^3}{b^3}$

**7)** $-1$

**8)** $-\dfrac{1}{2\sqrt{2}}$

**9)** $\sqrt{\mathrm{e}}$

**10)** $\dfrac{1}{\pi}$

**11)** $1$

**12)** $\dfrac{\sqrt{2}}{8x^3}$

**13)** $\dfrac{x^2}{2}$

**14)** $-\dfrac{3}{2}\left(x - \frac{\pi}{4}\right)$

**15)** $-x$

**Exercice 15 ( ✎ )** $\displaystyle\lim_{x\to 1} \frac{x^x - 1}{\ln x} = 1$

$\displaystyle\lim_{x\to 0}\left(\frac{x^2}{\ln(\cos x)} + \frac{2}{x^2}\sin^2 x\right) = 0$

$\displaystyle\lim_{x\to \frac{\pi}{4}} \frac{\ln(\sin^2 x)}{(\frac{\pi}{2}-x)^2} = -\frac{16\ln 2}{\pi^2}$

$\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\ln(\sin^2 x)}{(\frac{\pi}{2}-x)^2} = -1$

$\displaystyle\lim_{x\to +\infty} \sin\frac{1}{x}\tan\left(\frac{2\pi x}{4x+3}\right) = \frac{8}{3\pi}$

$\displaystyle\lim_{x\to 0^+} \ln x \tan(\ln(1+x)) = 0$

$\displaystyle\lim_{x\to \mathrm{e}} (\ln x)^{\tan\frac{\pi x}{2e}} = \mathrm{e}^{-\frac{2}{\pi}}$

**Exercice 17 ( ✎ )**

**1)** $\ln(\cos x) = -\dfrac{1}{2}x^2 - \dfrac{1}{12}x^4 - \dfrac{1}{45}x^6 + o\left(x^6\right)$

**2)** $\tan(x) = x + \dfrac{1}{3}x^3 + \dfrac{2}{15}x^5 + o\left(x^5\right)$

**3)** $\sin(\tan(x)) = x + \dfrac{1}{6}x^3 - \dfrac{1}{40}x^5 + o\left(x^5\right)$

**4)** $(\ln(1+x))^2 = x^2 - x^3 + \dfrac{11}{12}x^4 + o\left(x^4\right)$

**5)** $\exp(\sin(x)) = 1 + x + \dfrac{1}{2}x^2 + o\left(x^3\right)$

**6)** $\sin^6(x) = x^6 - x^8 + o\left(x^9\right)$

## Exercice 18 ( ✎ )

**1)** $\sqrt{x+1} = \sqrt{x} + \dfrac{1}{2\sqrt{x}} - \dfrac{1}{8x^{3/2}} + o\left(\dfrac{1}{x^{3/2}}\right)$ ;

**2)** $x\ln(x+1) - (x+1)\ln x = 1 - \ln(x) - \dfrac{1}{2x} + \dfrac{1}{3x^2} + o\left(\dfrac{1}{x^2}\right)$ ;

**3)** $\left(\dfrac{x+1}{x}\right)^x = \mathrm{e} - \dfrac{1}{2}\dfrac{\mathrm{e}}{x}\dfrac{1}{} + \dfrac{11}{24}\dfrac{\mathrm{e}}{x^2} + o\left(\dfrac{1}{x^2}\right)$ ;

**4)** $\operatorname{Arctan} x = \dfrac{\pi}{2} - \dfrac{1}{x} + \dfrac{1}{3x^3} + o\left(\dfrac{1}{x^4}\right)$.

## Exercice 19 ( ✎ )

**1)** $\dfrac{\arctan x - x}{\sin x - x} = 2 - \dfrac{11}{10}x^2 + o(x^2)$ ;

**2)** $\ln\sin x = \ln(\sqrt{2}/2) + (x - \pi/4) - (x-\pi/4)^2 + \dfrac{2}{3}(x-\pi/4)^3 + o((x-\pi/4)^3)$ ;

**3)** $(1+x)^{\frac{1}{x}} = e\left(1 - \dfrac{1}{2}x + \dfrac{11}{24}x^2 - \dfrac{7}{16}x^3\right) + o(x^3)$ ;

**4)** $x(\sqrt{x^2 + \sqrt{x^4+1}} - x\sqrt{2}) = \dfrac{\sqrt{2}}{8x^2} + o\left(\dfrac{1}{x^2}\right)$.

## Exercice 22 ( ✎ )

**1)** $\sqrt[3]{x^3 + x^2} - \sqrt[3]{x^3 - x^2} = x\left(\sqrt{1+\frac{1}{x}} - \sqrt{1-\frac{1}{x}}\right) = 1 + \dfrac{1}{8x^3} + o(1/x^3)$.

**2)** $\ln\left(\sqrt{1+x}\right) = \frac{1}{2}\ln x + \frac{1}{2x} + o(1/x)$.

## Exercice 23 ( ✎ )

**1)**  **a)** $\dfrac{\cos x}{\sqrt{1+x}} = 1 - \dfrac{x}{2} - \dfrac{x^2}{8} - \dfrac{x^3}{16} + \dfrac{49x^4}{384} + o(x^4)$

  **b)** $\dfrac{\sqrt{1+x}}{\cos x} = 1 + \dfrac{x}{2} + \dfrac{3x^2}{8} + \dfrac{5x^3}{16} + \dfrac{41x^4}{384} + o(x^4)$

  **c)** $\dfrac{\ln(1+x)}{\cos x} = x - \dfrac{x^2}{2} + \dfrac{5x^3}{6} - \dfrac{x^4}{2} + o(x^4)$

  **d)** $\dfrac{1 + \cos x}{2 + \sin x} = 1 - \dfrac{x}{2} + \dfrac{x^3}{12} - \dfrac{x^4}{16} + o(x^4)$

  **e)** $\dfrac{\sin(x/2)}{\mathrm{e}^{2x}} = \dfrac{x}{2} - x^2 + \dfrac{47x^3}{48} - \dfrac{5x^4}{8} + o(x^4)$

  **f)** $\dfrac{\ln(1+x)}{2 - \cos x} = x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + o(x^4)$

**2)**  **a)** $\dfrac{\sin(2x - \pi/4)}{\cos x} = 1 + 3(x - \dfrac{\pi}{4}) + \dfrac{3}{2}(x - \dfrac{\pi}{4})^2 + \dfrac{3}{2}(x - \dfrac{\pi}{4})^3 + \dfrac{19}{8}(x - \dfrac{\pi}{4})^4 + o((x - \dfrac{\pi}{4})^4)$

  **b)** $\dfrac{\cos(x-1)}{\ln(1+x)} = \dfrac{1}{\ln 2} - \dfrac{1}{2(\ln 2)^2}(x-1) + \dfrac{1}{\ln 2}\left(-\dfrac{1}{2} + \dfrac{1}{8\ln 2} + \dfrac{1}{4(\ln 2)^2}\right)(x-1)^2 + o((x-1)^2)$

  **c)** $\dfrac{\mathrm{e}^{x-1}}{\ln x} = \dfrac{1}{x-1} + \dfrac{3}{2} + \dfrac{11}{12}(x-1) + \dfrac{3}{8}(x-1)^2 + \dfrac{71}{720}(x-1)^3 + \dfrac{41}{1440}(x-1)^4 + o((x-1)^4)$

## Exercice 24

**1)** $\mathrm{e} - 1$

**2)** $-\dfrac{1}{2}$

**3)** $-\dfrac{e}{2}$

**4)** $\dfrac{1}{12}$

**5)** $-\dfrac{1}{6}$

**6)** $-\dfrac{1}{2}$